

Exercise 14

Consider the point (x, y) lying on the graph of $y = \sqrt{x - 3}$. Let L be the distance between the points (x, y) and $(4, 0)$. Write L as a function of y .

Solution

The distance from (x, y) to $(4, 0)$ is given by

$$\begin{aligned} L &= \sqrt{(4 - x)^2 + (0 - y)^2} \\ &= \sqrt{(4 - x)^2 + (-y)^2} \\ &= \sqrt{(4 - x)^2 + y^2}. \end{aligned} \tag{1}$$

Solve the given equation for x .

$$\begin{aligned} y &= \sqrt{x - 3} \\ y^2 &= x - 3 \\ y^2 + 3 &= x \end{aligned}$$

Therefore, equation (1) becomes

$$\begin{aligned} L &= \sqrt{[4 - (y^2 + 3)]^2 + y^2} \\ &= \sqrt{(4 - y^2 - 3)^2 + y^2} \\ &= \sqrt{(1 - y^2)^2 + y^2} \\ &= \sqrt{(1 - 2y^2 + y^4) + y^2} \\ &= \sqrt{y^4 - y^2 + 1}. \end{aligned}$$